Supplementary exercise 6.87 of IPS7e

Mean nicotine content in cigarettes. The advertised value is 1.4 mg. A study was carried out to determine whether the mean content was higher. The study wants to test the null hypothesis

$$\mathrm{H}_0 : \mu = 1.4\ mg$$

against the alternative hypothesis

$$\mathrm{H}_a : \mu > 1.4\ mg,$$

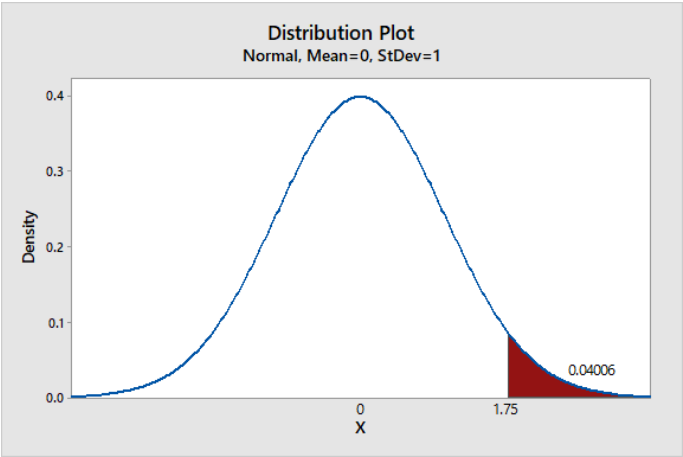
where μ is the mean nicotine content in the cigarette population of interest.

(a) A z-test statistic was computed: $z = 1.75$. (Note that we sometimes use the notation $z_{\text{obs}} = 1.75$ instead, when we want to stress the difference between the random variable z and the observed value (1.75); in the notation of the course we typically denote a random variable from $\mathrm{N}(0, 1)$ by Z.) This value is significant at the 5% level, because with a one-sided H_a we compute the P-value as $P = P(Z > 1.75) = 0.040$. For example, Table B of PSLS gives $P(Z < 1.75) = 0.9599$ and $P(Z < -1.75) = 0.0401$. Therefore, the P-value is indeed less than 0.05.

Alternatively, we could note that the 5% cut-point (sometimes called the critical value) for the test with this one-sided alternative is the 95% percentile of $\mathrm{N}(0, 1)$, with 95% to the left and 5% to the right. We can use the normal distribution table (with probability 0.95) or the table of critical values/percentiles (with upper tail probability 0.05) to determine the 95% percentile as 1.645. Because $z > 1.645$, the test is significant at the 5% significance level.

(b) We already computed the P-value, and it was greater than 0.01. Therefore, the test is not significant at the 1% level. Alternatively, the 1% cut-point for the test is the 99% percentile of $\mathrm{N}(0, 1)$, which we find in the tables as 2.326, and $z < 2.326$.

Finally, we show a Minitab `Probability Distribution Plot` to compute the tail probability, as well as some Minitab listings from the `Calc-Probability Distributions` menu.



Cumulative Distribution Function

Normal with mean = 0 and standard deviation = 1

x	P(X ≤ x)
1.75	0.959941

Inverse Cumulative Distribution Function

Normal with mean = 0 and standard deviation = 1

P(X ≤ x)	x
0.95	1.64485