Supplementary exercise 6.115 of IPS7e

Data: SAT scores of 500 high school students selected as a simple random sample from a certain population. Interest is in whether the population mean is larger than 450.

Model: A simple random sample (or i.i.d. observations) from a population with mean and standard deviation sigma, where sigma is known ($\sigma = 100$). In this situation, it might actually be reasonable to assume the standard deviation to be known because large groups of students have been tested; the assumption is then that this particular population has the same standard deviation as the more general population.

Test: The hypotheses of interest are:

$$\mathrm{H}_0 : \mu = 450 \quad \text{versus} \quad \mathrm{H}_a : \mu > 450.$$

The one-sided alternative hypothesis can perhaps be motivated by the interest being in whether this population performs better than 450; in any case, it is given in the question.

The test will be carried out at a 1% significance level (somewhat unusually). We should do a power calculation for a hypothesized true mean of $\mu = 460$, corresponding to a hypothesized true difference between true mean and tested mean of $460 - 450 = 10$. Although the wording of the question hints at a manual calculation, we will use software (the `Power and Sample Size` menu in Minitab). The question asks us to assume the population standard deviation to be known, so we need to use the `1-Sample Z` submenu. It would also be acceptable to assume the standard deviation to be estimated from the data (i.e., the `1-Sample t` submenu), and with the large sample size of 500 the difference in power would be minimal and negligible in practice.

```
Power;
 ZOne;
   Sample 500;
   Difference 10;
   Sigma 100;
   Alternative 1;
   Alpha 0.01.
```

Power and Sample Size

1-Sample Z Test
Testing mean = null (versus > null)
Calculating power for mean = null + difference
α = 0.01 Assumed standard deviation = 100

Results

Difference	Sample Size	Power
10	500	0.464032

The computed power is 0.46. This means that there is a 46% chance of rejecting the null hypothesis (i.e., $\mu = 450$) in favour of the alternative hypothesis (i.e., $\mu > 450$) based on a sample of 500 students, if the true mean was 460. With such a fairly low power, we need a bit of "luck" to get significance, and a non-significant result could easily happen by chance (even if the null hypothesis is false). It would be fair to say that the sample of 500 students has insufficient power. Part of the reason for this is the unusually low significance level; with the more standard $\alpha = 0.05$, the power becomes 0.72 (not shown in the listing).