Supplementary exercise 7.68 of IPS7e

(same data as in Supplementary Exercise 7.58 of IPS7e)

Data: 34 difference scores of preschool children in tests for spatial-temporal reasoning after and before attending piano lessons.

Model: the 34 observations are a simple random sample (i.i.d. sample) from a distribution corresponding to a population of difference scores for children that attend piano lessons. As we will use a nonparametric method, the population parameter of interest is the median.

Estimation: sample median = 4.0 (from Exercise 7.58).

95% CI for median: from the Minitab listing (next page) we take this interval to be $(3, 5)$ – use the interval with confidence level 95% obtained by interpolation.

Test: we split into three steps: hypothesis, test calculation and conclusion,

- we take the hypotheses as follows,
    - ∗ null hypothesis H_0: median = 0 (no improvement by the piano lessons),
    - ∗ alternative hypothesis H_a: median > 0 (some improvement).

  The hypotheses may also be stated in terms of the probability p of an improvement (that is, the (non-zero) difference after-before is positive):

$$H_0 : p = 0.5 \quad \text{versus} \quad H_a : p > 0.5.$$

- we carry out the test as a sign test using 28 positive and 4 negative differences, so that $P = P(Y \geq 28)$ where $Y \sim B(32, 0.5)$, and from Minitab/Stata analyses for the sign test we get an exact P-value assessed as $P < 0.0005$.
- we conclude that the test gives very strong evidence of a positive median in the distribution of differences. We can be pretty sure that the scores are more likely to be higher than lower on the second measurement (after the piano lessons).

As an additional note, our previous analyses in Exercises 7.58 and 7.59 did not identify any substantial problems with the normal distribution assumption. Therefore, the need for a non-parametric analysis is not obvious, and it is no surprise that our conclusions are very similar to those of the normal distribution analysis.

Further details on calculation of P-value

Because the test is carried out in the binomial distribution, we have several alternative ways of computing the P-value. First we can do an exact calculation of the probability in the $\text{Bin}(32, 0.5)$ distribution; the Minitab listing below gives $P = P(Y \geq 28) = P(Y \leq 4) = 0.0000097$.

We may also compute an approximate P-value using either the z-test for 1 proportion, or the normal approximation to B(32,0.5). These approaches are really only of interest without access to software (or in lucky cases a table for the binomial distribution in question).

The approximation of B(32,0.5) by a normal distribution gives:

$$P \approx 1 - P\left(Z < \frac{28 - 0.5 - 16}{\sqrt{32 \cdot 0.5 \cdot (1 - 0.5)}}\right) = 1 - P(Z < 4.066) = P(Z < -4.066) = 0.000024.$$

This computation involves a continuity-correction in the binomial distribution. The approximation is barely applicable because its condition is strictly speaking not met (because $32 \cdot 0.5 \cdot (1-0.5) = 8 < 10$), but the less strict condition given in the IPS and PSLS textbooks is met (specifically, $32 \cdot 0.5 = 16 > 10$ and $32 \cdot (1 - 0.5) = 16 > 10$).

The classical z-test for 1 proportion gives:

$$P \approx P\left(Z > \frac{28/32 - 0.5}{\sqrt{0.5 \cdot 0.5/32}}\right) = P(Z > 4.243) = P(Z < -4.243) = 0.000011.$$

The conditions for use of this test are met (specifically, $32 \cdot 0.5 = 16 > 10$ and $32 \cdot (1-0.5) = 16 > 10$). This calculation really amounts to computing the tail probability without a continuity correction (adding/subtracting 0.5) in the binomial distribution, and in this case the resulting value is a better approximation to the true value; in most cases the calculation with a continuity correction will be closest. In any case, we see that here our use of any of these approximations will in no way affect the conclusion.

We finally show the relevant Minitab commands (in the order we have referred to them above), and the resulting output.

```
SInterval 95.0 'change'.
STest 0.0 'change';
 Alternative 1.
CDF 4;
 Binomial 32 .5.
POne 32 28;
 Test .5;
 Confidence 95.0;
 Alternative 1.
```

Sign CI: change

Method

η: median of change

Descriptive Statistics

Sample	N	Median
change	34	4

95% Confidence Interval for η

Sample	CI for η	Achieved Confidence	Position
change	(3, 5)	94.24%	(12, 23)
	(3, 5)	95.00%	Interpolation
	(3, 5)	97.57%	(11, 24)

Sign Test for Median: change

Method

η: median of change

Descriptive Statistics

Sample	N	Median
change	34	4

Test

Null hypothesis H_0: η = 0
Alternative hypothesis H_1: η > 0

Sample	Number < 0	Number = 0	Number > 0	P-Value
change	4	2	28	0.000

Cumulative Distribution Function

Binomial with n = 32 and p = 0.5

x	P(X ≤ x)
4	0.0000097

Test and CI for One Proportion

Method

p: event proportion
Exact method is used for this analysis.

Descriptive Statistics

N	Event	Sample p	95% Lower Bound for p
32	28	0.875000	0.736403

Test

Null hypothesis H_0: p = 0.5
Alternative hypothesis H_1: p > 0.5

P-Value
0.000