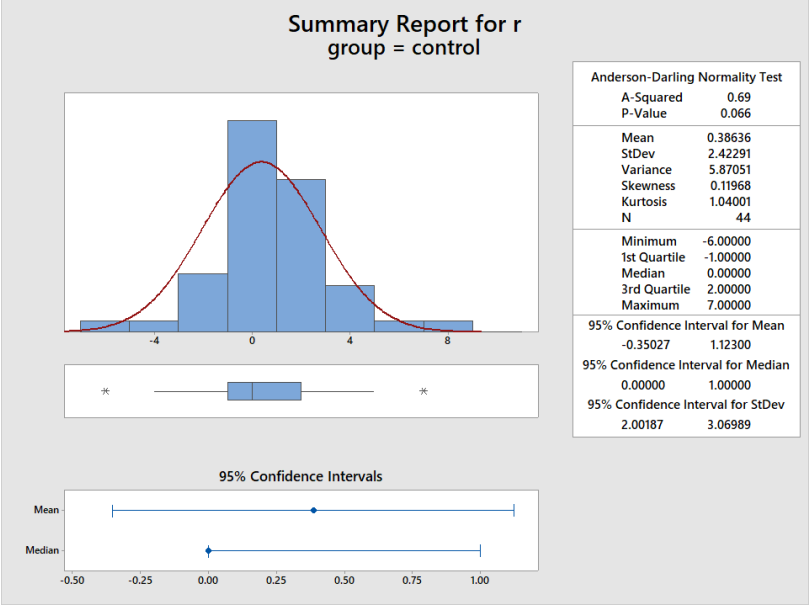
Supplementary exercises 7.102, 7.103 and 7.104 of IPS7e

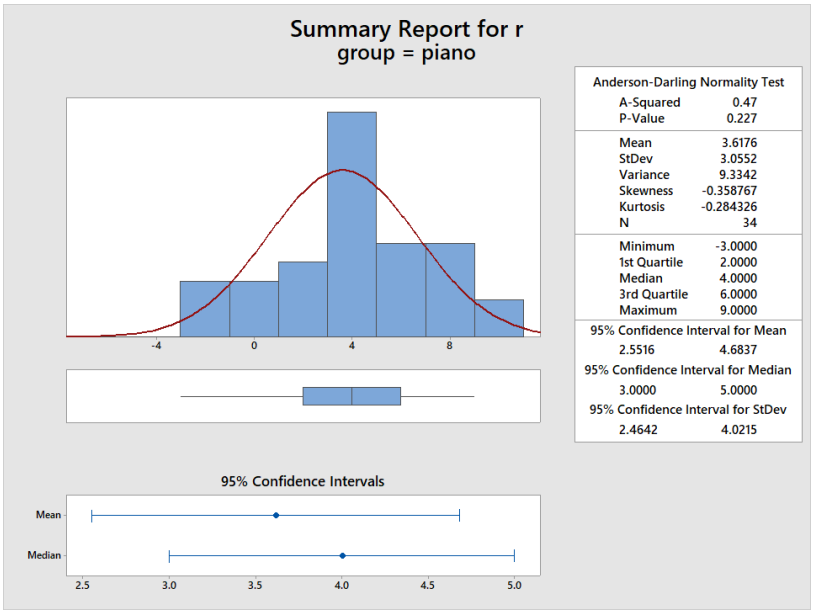
Data: 2 samples of changes (improvements, differences after-before) in spatial-temporal reading test scores for 34 children attending six months of piano lessons and 44 children in a control group. Note that we already analyzed the data for the piano group alone in Exercises 7.58 and 7.59.

Model: the 2 samples are independent and each a simple random sample (i.i.d. sample) from a distribution with unknown mean and standard devation (μ_1 and σ_1 for the piano lesson group; μ_2 and σ_2 for the control group).

Exercise 7.103, (a)+(b)
First, some Minitab commands and output for statistics and graphs of interest for the descriptive analysis. In the data file, the variable containing the differences has somewhat mysteriously been labeled as "r", but we choose to not change it here.

```
GSummary 'r';
 By 'group'.
Stem-and-Leaf 'r';
 By 'g'.
Dotplot ( 'r' ) * 'group'.
PPlot 'r';
 Normal;
 Symbol;
 FitD;
 Grid 2;
 Grid 1;
 MGrid 1;
 Panel 'group'.
```





Stem-and-leaf of r g = 0 N = 34

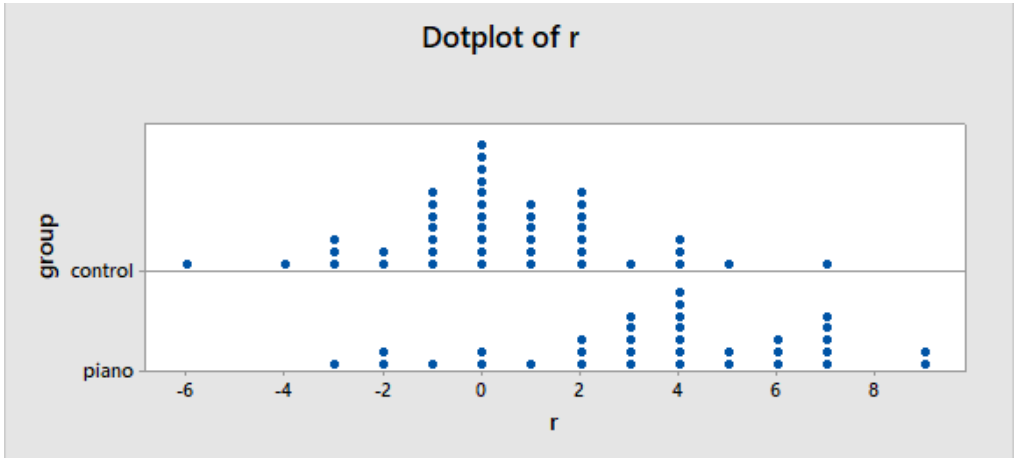
1	-3	0
3	-2	00
4	-1	0
4	-0	
6	0	00
7	1	0
10	2	000
15	3	00000
(7)	4	0000000
12	5	00
10	6	000
7	7	00000
2	8	
2	9	00

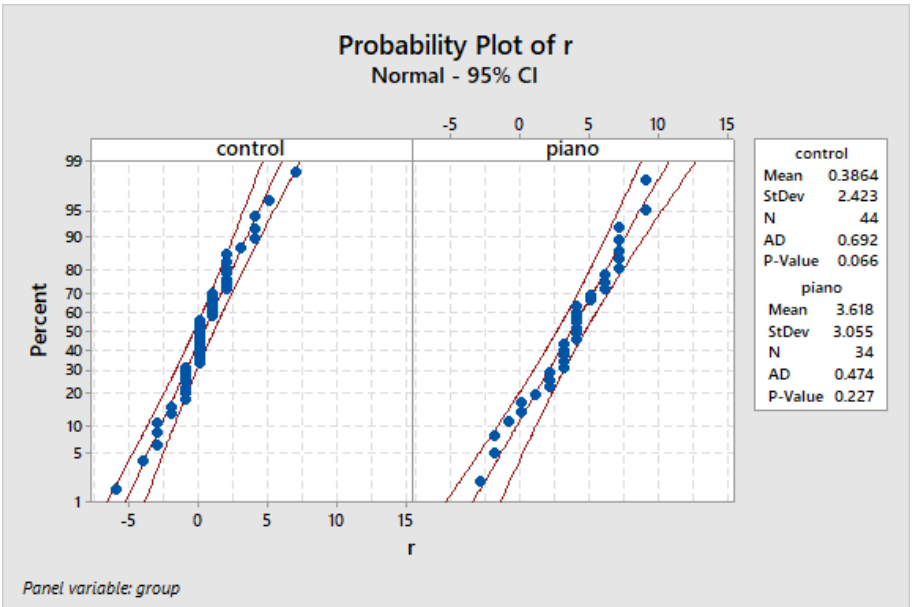
Leaf Unit = 0.1

Stem-and-leaf of r g = 1 N = 44

1	-6	0
1	-5	
2	-4	0
5	-3	000
7	-2	00
14	-1	0000000
14	-0	
(11)	0	00000000000
19	1	000000
13	2	0000000
6	3	0
5	4	000
2	5	0
1	6	
1	7	0

Leaf Unit = 0.1





Comments:
The distributions are best displayed by stemplots and dotplots. Note that the stemplot in Minitab artificially divides the observations with a value of zero into 2 groups (this is clearly not desirable). The listing of descriptive statistics contains the mean, standard deviation and standard error of the mean, as requested, plus many more.

Both distributions look reasonably symmetric and bell-shaped. The discreteness of the values is clearly seen in both the dotplots and the normal probability plots. Nevertheless, the normal plots and normality tests show no reason to reject a normal distribution for the piano group. The distribution for the control group is somewhat too peaked for a normal distribution (kurtosis $= 1.04$), and the P-value for the normality is as low as $P = 0.066$. Among the different normality tests, the A-D test is the only one showing something near significance for the control group. It seems reasonable to maintain a normal distribution assumption even in view of a possible mild violation.

Exercise 7.102, (c) and Exercise 7.103
The interest is in comparing the changes in score between the piano and control groups. Even if the primary interest is in an improvement of the scores in the piano group over the control group, there seems to be no apriori reason to focus only on an improvement in the score. Therefore, our hypotheses are

$$\mathrm{H}_0 : \mu_1 = \mu_2 \quad \text{versus} \quad \mathrm{H}_a : \mu_1 \neq \mu_2.$$

Because both distributions look reasonably normal, we may assume normal distributions and obtain exact inference (confidence interval and test). Minitab commands and output:

```
TwoT 'r' 'group';
 Confidence 95.0;
 Test 0.0;
 Alternative 0.
```

Two-Sample T-Test and CI: r, group

Method

μ_1: mean of r when group = control
μ_2: mean of r when group = piano
Difference: $\mu_1 - \mu_2$

Equal variances are not assumed for this analysis.

Descriptive Statistics: r

group	N	Mean	StDev	SE Mean
control	44	0.39	2.42	0.37
piano	34	3.62	3.06	0.52

Estimation for Difference

Difference	95% CI for Difference
-3.231	(-4.508, -1.954)

Test

Null hypothesis $H_0: \mu_1 - \mu_2 = 0$
Alternative hypothesis $H_1: \mu_1 - \mu_2 \neq 0$

T-Value	DF	P-Value
-5.06	61	0.000

Comments:
The t-test (without assuming same standard deviations in the two groups) gives a value of $t = 5.06$ with approximate df $= 61$, and this is highly significant; we could for example state this as $P < 0.001$. There is clear evidence of a difference in scores in the two groups: the piano lesson group scored higher than the control group. We also note that the evidence against H_0 is so strong that any deviations from the normal distribution are without practical importance. The 95% confidence interval gives the range of the improvement as about 2 to 4.5 units (of test scores).

Finally, a Minitab technical note: The difference between the two group means is computed as control minus piano, and therefore shows as negative. If we were interested in the difference piano minus control, we can use all the above results and simply switch the signs. Alternatively, we can make Minitab do the difference in the preferred way by changing the labels for the groups so that the piano group becomes the first one (alphabetically, it's the second one). The variable "g" in the Minitab worksheet would do this (`g=0` for piano, `g=1` for control). We could also unstack the two columns and then enter the columns in the desired order.

Exercise 7.104
The advantage of including a control group is that any improvement in the scores by aging (or perhaps other types of confounding) is taken into account. The control group data show that such an improvement is at most minor.

The primary advantage of carrying out a significance test over using a confidence interval is that it gives a P-value, which is a more informative measure of the evidence against the null hypothesis than mere significance at 5% level. On the other hand, the confidence interval is useful, actually indispensable, to quantify the likely magnitude of the effect; recall that statistical significance is not the same as biological significance.