Supplementary exercises 6.13 and 6.14 of IPS7e

Exercise 6.13

Yields of corn in bushels per acre. Let $X_1, \ldots, X_{15}$ denote the 15 yields, i.e. $n = 15$. We assume that the observations are i.i.d. (independent and identically distributed) with mean μ and standard deviation σ; as usual, they are the unknown population parameters corresponding to the population these yields of corn could be representative for. We also assume (quite unrealistically) that σ is known to be $\sigma = 10$. The average of the X's is: $\bar{X} = 123.80$.

(a) With a confidence level of 90%, our z^*-value is the 95% percentile of N(0,1), which equals $z^* = 1.645$. Therefore,

$$90\% \text{ CI}: \ \bar{X} \pm z^*\sigma/\sqrt{n} = 123.8 \pm 1.645 \cdot 10/\sqrt{15} = 123.8 \pm 4.2 = (119.6, 128.0).$$

Minitab command and output for this calculation (from the "Basic Statistics – 1-Sample Z" menu):

```
OneZ 'Yield';
 Sigma 10;
 Confidence 90;
 Alternative 0.
```

One-Sample Z: Yield

Descriptive Statistics

N	Mean	StDev	SE Mean	90% CI for μ
15	123.80	12.26	2.58	(119.55, 128.05)

μ: mean of Yield
Known standard deviation = 10

(b) For a 95% confidence level, we will instead use $z^* = 1.96$:

$$95\% \text{ CI}: \ \bar{X} \pm z^*\sigma/\sqrt{n} = 123.8 \pm 1.96 \cdot 10/\sqrt{15} = 123.8 \pm 5.1 = (118.7, 128.9).$$

(c) With a confidence level of 99%, our z^*-value is the 99.5% percentile of N(0,1), which equals $z^* = 2.576$:

$$99\% \text{ CI}: \ \bar{X} \pm z^*\sigma/\sqrt{n} = 123.8 \pm 2.576 \cdot 10/\sqrt{15} = 123.8 \pm 6.7 = (117.1, 130.5).$$

(d) The margins of error increase with increasing confidence levels.

Minitab listings for parts (b) and (c):

One-Sample Z: Yield

Descriptive Statistics

N	Mean	StDev	SE Mean	95% CI for μ
15	123.80	12.26	2.58	(118.74, 128.86)

μ: mean of Yield
Known standard deviation = 10

One-Sample Z: Yield

Descriptive Statistics

N	Mean	StDev	SE Mean	99% CI for μ
15	123.80	12.26	2.58	(117.15, 130.45)

μ: mean of Yield
Known standard deviation = 10

Exercise 6.14

We assume (hypothetically) that the $\bar{X} = 123.80$ came from a sample of size $n = 50$.

(a) For a 95% confidence level, our $z^* = 1.96$ and it is therefore the same formula as in (b) of the preceding exercise:

$$95\% \text{ CI}: \ \bar{X} \pm z^* \sigma/\sqrt{n} = 123.8 \pm 1.96 \cdot 10/\sqrt{50} = 123.8 \pm 2.8 = (121.0, 126.6).$$

(b) The margin of error for $n = 50$ (which we calculated as 2.8) is narrower than the margin of error for $n = 15$ (5.1). This is because a larger sample size provides more precise information about the unknown parameter (here, the population mean). Generally speaking, margin of errors always decrease with increasing sample size.

(c) The margin of errors for 90% and 99% intervals will also be narrower for $n = 50$ than for $n = 15$, for the reasons just explained above. The ratio between the margin of errors obtained at sample sizes 15 and 50 will be $\sqrt{50}/\sqrt{15} = 1.825$, so that the interval for $n = 15$ is 1.825 times wider than that for $n = 50$, regardless of the confidence level.

Note that we can also use Minitab for the calculation in (a), by typing in the relevant value (the sample mean of 123.8) as “Summarized data” instead of using the data in the column of the worksheet.

One-Sample Z

Descriptive Statistics

N	Mean	SE Mean	95% CI for μ
50	123.80	1.41	(121.03, 126.57)

μ: mean of Sample
Known standard deviation = 10